Macro Theory B

Final exam (spring 2014) Second Date - Solution

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1 Economy with tax evasion

1. The state variables for the household are the levels of assets saved from the previous period $a_{i,t-1}$, the amount of tax evaded in the previous period $x_{i,t-1}$, preference for leisure $\psi_{i,t}$ and whether the household is audited this period $I_{i,t}$.

2. The household problem:

$$
\begin{aligned}
V\left(a_{t-1}, x_{t-1}, \psi_t, I_{i,t}\right) &= \max_{c_t,h_t,a_t,\phi_t} \left\{u\left(c_t\right) + \psi_t\left(1-h_t\right) + \beta E_t V\left(a_t, x_t, , \psi_{t+1}, I_{t+1}\right)\right\} \\
& \quad s.t. \\
a_t &\geq 0 \\
\phi_t &\in [0,1] \\
c_t + a_t + I_t z(x_{t-1}) &\leq w_t h_t + (1+r)a_{t-1} - \tau\phi_t r a_{t-1} + b \\
x_t &= (1-\phi_t)\tau r a_{t-1} \\
E I_{t+1} &= \pi \\
& \quad \text{and the stochastic processof } \psi
\end{aligned}
$$

The only aggregate state variable is K_t.

3. The stationary recursive equilibrium is:

- A set of prices w,r
- A monitoring probability π
- A tax rate τ
- Lum sum transfer b
- A decision function for the households $[c_t, h_t, a_t, \phi_t] = g\left(a_{t-1}, x_{t-1}, \psi_t, I_{i,t}, r, w\right)$
- A decision function for the firms how much capital and labor to employ
- The household decision function is optimal given the prices and the constrains
- The firm decision function is optimal given the prices
- The aggregate resource constrain holds: $C_t + K_{t+1} - (1-\delta)K_t = AK_t^{\alpha} H_t^{1-\alpha}$
- The government budget is balanced: $b = \int \tau\phi_i r a_{i,t-1}$

4. The government problem:

$$
\begin{aligned}
& \max_{\pi} \int \left[u\left(ci\right) + \psi_i\left(1-h_i\right)\right] \\
& s.t. \\
b &= \int \tau \phi_i r a_{i,t-1} - m(\pi) \\
C_t + K_{t+1} - (1-\delta)K_t &= AK_t^{\alpha} H_t^{1-\alpha} \\
& \text{And the competitive equilibrium that } \pi \text{ will yield}
\end{aligned}
$$

The governemnt tradeoffs here are about redistribution. If all the households were equel (ex post), there is no point in taxing the households for the lump sum transfer. Assuming that τ is exoganous, it would be optimal for the government to set $\pi = 0$. In this case there is effectively no capital gain taxes which increases output and welfare. However, as there is a (potentially persistence) process for $\psi_{i,t}$, there is a distribution of assets, income and welfare in the economy. The more disperse is the distribution, the higher is the benefit for the benevolent government to tax and distribute.

2 Search and matching

1. The value functions of the revised model:

Employed workers, who recieve the compansation package in their first period of unemployment:

$$W_t = w_t + \beta\left[\sigma U_{t+1} + \sigma(w-b) + (1-\sigma)W_{t+1}\right] \tag{1}$$

Unemployed workers:

$$U_t = b + \beta\left[(1-\theta_t q(\theta_t))U_{t+1} + \theta_t q(\theta_t)W_{t+1}\right] \tag{2}$$

The value of a vacancy:

$$V_t = -\xi + \beta\left[1-q(\theta_t))V_{t+1} + q(\theta_t)J_{t+1}\right] \tag{3}$$

And with free entry:

$$J_{t+1} = \frac{\xi}{\beta q(\theta_t)} \tag{4}$$

The value of a filled position, already assuming $V = 0$:

$$J_t = p - w + \beta\left[-\sigma(w-b) + (1-\sigma)J_{t+1}\right] \tag{5}$$

From the bargaining problem:

$$W_t - U_t = \gamma\left(J_t + W_t - U_t\right) \tag{6}$$

2. In a steady state equilibrium, from (5):

$$\begin{aligned} J &= p - w + \beta\left[-\sigma(w-b) + (1-\sigma)J\right] && (7) \\ p - w - \beta\sigma(w-b) &= J(1-\beta(1-\sigma)) && (8) \end{aligned}$$

plugging (4):

$$\begin{aligned} p - w - \beta\sigma(w-b) &= \frac{(1-\beta(1-\sigma))\xi}{\beta q(\theta)} && (9) \\ p - w - \beta\sigma(w-b) &= \frac{(\frac{1-\beta}{\beta} + \sigma)\xi}{q(\theta)} && (10) \end{aligned}$$

Or:

$$p - w - \beta\sigma(w-b) = \frac{(r+\sigma)\xi}{q(\theta)} \qquad (11)$$

From (1):

$$W = \frac{w + \beta\sigma U + \beta\sigma(w-b)}{1-\beta(1-\sigma)} \qquad (12)$$

Plugging (7) and (12) into (6):

$$\begin{aligned} & \frac{w + \beta\sigma U + \beta\sigma(w-b)}{1-\beta(1-\sigma)} - U \\ = \; & \gamma\left(\frac{p - w - \beta\sigma(w-b)}{1-\beta(1-\sigma)} - U + \frac{w + \beta\sigma U + \beta\sigma(w-b)}{1-\beta(1-\sigma)} - U\right) \\ & w + \beta\sigma U + \beta\sigma(w-b) - U(1-\beta(1-\sigma)) \\ = \; & \gamma\left(p - w - \beta\sigma(w-b) + w + \beta\sigma U + \beta\sigma(w-b) - U(1-\beta(1-\sigma))\right) \end{aligned}$$

Or:

$$w + \beta\sigma(w-b) - U(1-\beta) = \gamma\left(p - U(1-\beta)\right) \qquad (13)$$

From (2):

$$U(1-\beta) = b + \beta\theta q(\theta)\left[W - U\right] \qquad (14)$$

From (6) and (4):

$$W-U=\frac{\gamma}{1-\gamma}\frac{\xi}{\beta q(\theta)} \tag{15}$$

Combining:

$$U(1-\beta)=b+\beta\theta q(\theta)\frac{\gamma}{1-\gamma}\frac{\xi}{\beta q(\theta)}=b+\frac{\gamma}{1-\gamma}\theta\xi \tag{16}$$

Plugging (16) into (13):

$$\begin{aligned} w+\beta\sigma(w-b)-b-\frac{\gamma}{1-\gamma}\theta\xi &= \gamma\left(p-b-\frac{\gamma}{1-\gamma}\theta\xi\right) &(17)\\ w+\beta\sigma(w-b) &= b+\gamma\left(p-b-\frac{\gamma}{1-\gamma}\theta\xi+\frac{1}{1-\gamma}\theta\right) &(18)\\ w+\beta\sigma(w-b) &= b(1-\gamma)+\gamma\left(p+\theta\xi\right) &(19)\end{aligned}$$

Combining (11) and (19) we get the solution:

$$p-b(1-\gamma)-\gamma\left(p+\theta\xi\right)=\frac{(r+\sigma)\xi}{q(\theta)} \tag{20}$$

3. As can be seen from teh solution to #2, the stady state equation for the market tightness θ is the same as in the textbook case. It can be easily seen from the steady state equation that this also yields the same solution for J,W,U and also v,u. The only departure will be the employment period wage, w. This means that the severance payment did not increase the value for the workers - given that the combind productivity of the match was not altered, and that the bargainig power is still the same, the new law only changes the schedule of the payments from the firm to the worker, by lowering the per period wage and adding another payment at the end.

3 Quadratic utility

1. Certainty equivalence is where higher moments of the distribution do not matter for the solution. i.e. we will get the same solution for a stochastic process where $Ey_t=\bar{y}$ and for a determoinistic process where $y_t=\bar{y}$.

2. The Euler condition is:

$$u_c=\beta(1+r)Eu_c^{'}$$

and with the question parameters and utility function:

$$b_1-b_2c_t=E\left[b_1-b_2c_{t+1}\right]$$

Or:

$$E\left[c_{t+1}\right] = c_t$$

Using the law of iterated expectations:

$$E_t\left[c_{t+2}\right] = E_t\left[E_{t+1}\left[c_{t+2}\right]\right] = E_t\left[c_{t+1}\right] = c_t$$

And more generaly:

$$E_t\left[c_{t+j}\right] = c_t \quad \forall \quad j \geq 0$$

Working with the budget constraint:

$$\begin{aligned}
a_{t+1} &= (1+r)(a_t + y_t - c_t) \quad \Leftrightarrow \quad a_t = \frac{1}{1+r}a_{t+1} + c_t - y_t \\
a_t &= c_t - y_t + \frac{1}{1+r}E_t\left[c_{t+1} - y_{t+1} + \frac{1}{1+r}a_{t+2}\right] = \\
&= c_t - y_t + E_t\left[\frac{c_{t+1} - y_{t+1}}{1+r} + \frac{1}{(1+r)^2}\left[c_{t+2} - y_{t+2} + \frac{1}{1+r}a_{t+3}\right]\right] \\
&= ... = E_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{t+j}(c_{t+j} - y_{t+j}) + E_t\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1}a_{t+j+1}
\end{aligned}$$

Or:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}E_tc_{t+j} = a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}E_ty_{t+j} - E_t\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^{j+1}a_{t+j+1}$$

Removing the last term due to the "no Ponzi scheme" condition:

$$\Rightarrow \quad \begin{aligned}
c_t\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j} &= a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}E_ty_{t+j} \\
c_t &= \frac{r}{1+r}\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}E_ty_{t+j}\right]
\end{aligned}$$

And as we can see the consumption equals the annuity value of the total wealth. We can also see that the certainty equivalence holds as only the first moment of the expected future income matters.

3. The properties that are required for this result are the linear marginal utility of consumption to get the martingale behavior for consumption, and the given martingale process of the income